HOMEWORK SET 8: 3-D SCHRÖDINGER EQUATION II
Due Monday, February 17, 2025

PROBLEM FROM AOD

The general 3-dimensional Schrödinger Equation in spherical cooridinates is (equation 8.49 with substitutions)

$$a_B = \frac{\hbar^2}{m_e k e^2} \qquad E = -\frac{E_R}{n^2} = -\frac{m_e\left(ke^2\right)^2}{2\hbar^2 n^2} = -\frac{ke^2}{2a_B n^2}$$

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2} = \left[\frac{1}{a_B^2 n^2} - \frac{2}{a_B r}\right]\psi$$

a) Using the tables in the text (8.2 below, 8.1 on HW Set 6) write out the separate solutions for $R_{2,1}(r)$, $\Theta_{1,-1}(\theta)$, and $\Phi_{-1}(\phi)$ then write out $\psi_{2,1,-1}(r,\theta,\phi)$. (COMBINE CONSTANTS INTO A … WRITE WHAT A IS!)

b) Show that the $\psi_{2,1,-1}(r,\theta,\phi)$ you've written down is a solution to the 3-D Schrödinger equation. (HINT: EVALUATE EACH TERM ON THE LEFT SEPARATELY, **SIMPLIFY THEM**, THEN ADD THEM TOGETHER. SUBSTITUTE ψ ON THE RIGHT, CANCEL COMMON TERMS AND DO THE ALGEBRA TO GET 1 = 1 OR 0 = 0)

	n = 1	n = 2	n = 3
ℓ = 0	$R_{1,0} = \frac{2}{\sqrt{a_B^3}}e^{-r/2a_B}$	$R_{2,0} = \frac{1}{\sqrt{2a_B^3}}\left(1-\frac{r}{2a_B}\right)e^{-r/2a_B}$	$R_{3,0} = \frac{2}{\sqrt{27a_B^3}}\left(1-\frac{2r}{3a_B}+\frac{2r^2}{27a_B^2}\right)e^{-r/3a_B}$
ℓ = 1		$R_{2,1} = \frac{1}{\sqrt{24a_B^5}}re^{-r/2a_B}$	$R_{3,1} = \frac{8}{27\sqrt{6a_B^5}}\left(1-\frac{r}{6a_B}\right)re^{-r/3a_B}$
ℓ = 2			$R_{3,2} = \frac{4}{81\sqrt{30a_B^7}}r^2e^{-r/3a_B}$